التمرين الأول :

معادلات حركة القذيفة تتم في المستوي (oxy) فإن :

$$\vec{r} = x\vec{e_x} + y\vec{e_y} \Rightarrow \vec{v} = \frac{d\vec{r}}{dt} = v_x\vec{e_x} + v_y\vec{e_y}$$

$$m\frac{d^2\vec{r}}{dt^2} = m\frac{d\vec{v}}{dt} = -mg\vec{e_y} - Bv(v_x\vec{e_x} + v_y\vec{e_y})$$

$$\Rightarrow \begin{cases} \frac{d^2x}{dt^2} = -\frac{B}{m}v\,v_x & \text{--- (I)} \\ \frac{d^2y}{dt^2} = -g - \frac{B}{m}v\,v_y & \text{--- (II)} \end{cases}$$

0.1

خوارزمية رونج-كوتا للمعادلة (I) :

$$\begin{cases} \frac{dx}{dt} = v_x \\ \frac{dv_x}{dt} = -\frac{B}{m}v\,v_x \end{cases}$$

$$\frac{x_{i+1} - x_i}{\Delta t} = v_{x_{i+1/2}} \Rightarrow x_{i+1} = x_i + \Delta t\, v_{x_{i+1/2}} = x_i + K_2$$

0.1

$$\frac{x_{i+1/2} - x_i}{\Delta t/2} = v_{x_i} \Rightarrow x_{i+1/2} = x_i + \frac{1}{2}\Delta t\, v_{x_i} = x_i + \frac{K_1}{2}$$

$$\frac{v_{x_{i+1}} - v_{x_i}}{\Delta t} = -\frac{B}{m}v_{i+1/2}v_{x_{i+1/2}} \Rightarrow v_{x_{i+1}} = v_{x_i} - \frac{B}{m}v_{i+1/2}v_{x_{i+1/2}}\cdot\Delta t$$
$$= v_{x_i} + K_4$$

$$\frac{v_{x_{i+1/2}} - v_{x_i}}{\Delta t/2} = -\frac{B}{m}v_i v_{x_i} \Rightarrow v_{x_{i+1/2}} = v_{x_i} - \frac{1}{2}\Delta t \times \frac{B}{m}v_i v_{x_i}$$
$$= v_{x_i} + \frac{1}{2}K_3$$

خوارزمية رونج-كوتا للمعادلة (II)

$$\begin{cases} \frac{dy}{dt} = v_y \\ \frac{dv_y}{dt} = -g - \frac{B}{m}v\,v_y \end{cases}$$

بنفس الطريقة نجد أن :

$$y_{i+1} = y_i + \Delta t\, v_{y_{i+1/2}} = y_i + k'_2$$

$$y_{i+1/2} = y_i + \frac{1}{2}\Delta t\, v_{y_i} = y_i + \frac{1}{2} k'_2$$

$$v_{y_{i+1}} = v_{y_i} - \Delta t \left[g + \frac{B}{m} v_{i+1/2} v_{y_{i+1/2}}\right] = v_{y_i} + k'_4$$

$$v_{y_{i+1/2}} = v_{y_i} - \frac{1}{2}\Delta t \left[g + \frac{B}{m} v_i v_{y_i}\right] = v_{y_i} + \frac{1}{2} k'_3$$

ومنه خوارزمية رونج كوتا المرتبة الثانية تكتب :

$$v_i = \sqrt{v_{x_i}^2 + v_{y_i}^2}$$

$$k_3 = -\Delta t \frac{B}{m} v_i v_{x_i}$$

$$k'_3 = -\Delta t \left[g + \frac{B}{m} v_i v_{y_i}\right]$$

$$k_2 = -\Delta t \left[v_{x_i} + \frac{1}{2} k_3\right]$$

$$k'_2 = -\Delta t \left[v_{y_i} + \frac{1}{2} k'_3\right]$$

$$k_4 = -\Delta t \frac{B}{m} \sqrt{\left(v_{x_i} + \frac{1}{2}k_3\right)^2 + \left(v_{y_i} + \frac{1}{2}k'_3\right)^2}\left(v_{x_i} + \frac{1}{2}k_3\right)$$

$$k'_4 = -\Delta t \left[g + \frac{B}{m} \sqrt{\left(v_{x_i} + \frac{1}{2}k_3\right)^2 + \left(v_{y_i} + \frac{1}{2}k'_3\right)^2}\left(v_{y_i} + \frac{1}{2}k'_3\right)\right]$$

$$x_{i+1} = x_i + k_2$$

$$y_{i+1} = y_i + k'_2$$

$$v_{x_{i+1}} = v_{x_i} + k_4$$

$$v_{y_{i+1}} = v_{y_i} + k'_4$$

التمرين الثالث :

1- تعيين $P(x)$:

$$P(x) = \sum_{i=0}^{2} \lambda_i(x) f(x_i)$$

حيث أن

$$\lambda_i(x) = \prod_{\substack{j=0 \\ (i \neq j)}}^{2} \frac{x - x_j}{x_i - x_j}$$

$$\lambda_0(x) = \prod_{\substack{j=0 \\ j\neq 0}}^{2} \frac{x-x_j}{x_0-x_j} = \frac{x-x_1}{x_0-x_1} \times \frac{x-x_2}{x_0-x_2} = \frac{x-4}{1-4} \times \frac{x-9}{1-9}$$

$$= \frac{1}{24}\left[x^2-13x+36\right]$$

$$\lambda_1(x) = \frac{x-x_0}{x_1-x_0} \times \frac{x-x_2}{x_1-x_2} = \frac{x-1}{4-1} \times \frac{x-9}{4-9}$$

$$= -\frac{1}{15}\left[x^2-10x+9\right]$$

$$\lambda_2(x) = \frac{x-x_0}{x_2-x_0} \times \frac{x-x_1}{x_2-x_1} = \frac{x-1}{9-1} \times \frac{x-4}{9-4}$$

$$= \frac{1}{40}\left[x^2-5x+4\right]$$

$$P(x) = \lambda_0(x) f(x_0) + \lambda_1(x) f(x_1) + \lambda_2(x) f(x_2)$$

$$= \frac{1}{48}\cdot(x^2-13x+36) - \frac{1}{15}(x^2-10x+9)$$

$$+ \frac{3}{2}\times\frac{1}{40}\left[x^2-5x+4\right]$$ 05

$$= \left[\frac{1}{48} - \frac{1}{15} + \frac{3}{80}\right]x^2 + \left[\frac{-13}{48} + \frac{10}{15} - \frac{15}{80}\right]x$$

$$+ \left[\frac{36}{48} - \frac{9}{15} + \frac{12}{80}\right]$$

$$= \frac{10-32+18}{480}x^2 + \frac{-13\times10+10\times32-15\times6}{480}x$$

$$+ \frac{36\times10-9\times32+12\times6}{480}$$

$$= -\frac{1}{120}x^2 + \frac{5}{24}x + \frac{3}{10}$$ 02

التمرين الثالث :

1 - حساب $\langle S_i \rangle$:

$$\langle x_N \rangle = \frac{1}{K}\sum_{j=1}^{K} x_N^j$$

$$= \frac{1}{K}\sum_{j=1}^{K}\sum_{i=1}^{N} S_i^j = \sum_{i=1}^{N}\frac{1}{K}\sum_{j=1}^{K} S_i^j$$

$$= \sum_{i=1}^{N}\langle S_i \rangle$$

$$j=1 \Rightarrow x_4^1 = S_1^1 + S_2^1 + S_3^1 + S_4^1 = a + a - a + a = 2a$$

$$j=2 \Rightarrow x_4^2 = S_1^2 + S_2^2 + S_3^2 + S_4^2 = -a - a - a + a = -2a$$

$$j=3 \Rightarrow x_4^3 = S_1^3 + S_2^3 + S_3^3 + S_4^3 = a + a + a + a = 4a$$

$\Rightarrow$ 0.1 $\langle S_1 \rangle = \frac{1}{3}\sum_{j=1}^{3} S_1^j = \frac{1}{3}\left(S_1^1 + S_1^2 + S_1^3\right) = \frac{1}{3}(a - a + a) = \frac{a}{3}$

0.1 $\langle S_2 \rangle = \frac{1}{3}\sum_{j=1}^{3} S_2^j = \frac{1}{3}\left(S_2^1 + S_2^2 + S_2^3\right) = \frac{1}{3}(a - a + a) = \frac{a}{3}$

0.1 $\langle S_3 \rangle = \frac{1}{3}\sum_{j=1}^{3} S_3^j = \frac{1}{3}\left(S_3^1 + S_3^2 + S_3^3\right) = \frac{1}{3}(-a - a + a) = \frac{-a}{3}$

0.1 $\langle S_4 \rangle = \frac{1}{3}(a + a + a) = \frac{3}{3}a = a$

0.1 $\langle x_N \rangle = \sum_{i=1}^{4}\langle S_i \rangle = \frac{a}{3} + \frac{a}{3} - \frac{a}{3} + a = \frac{4}{3}a$